

A green vehicle routing problem with picking up and delivery based on an improved firefly algorithm

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Abstract

Transportation which plays an essential role in distribution impacts climate change and global warming. So, one of the critical problems in distribution is the green vehicle routing problem. This paper investigates a green vehicle routing problem with picking up and delivery and soft time windows for a single depot. The problem is developed as a single-objective mixed integer nonlinear programming model. The model aims to minimize fuel costs based on vehicle load and penalty costs for violating time windows while satisfying customer pickup and delivery demands simultaneously. This paper proposes an improved firefly algorithm based on chaotic initialization and differential evolution to solve the model. Results show that the proposed model minimizes carbon emissions while maximizing customer satisfaction. Compared with a traditional heuristic algorithm, the improved firefly algorithm has higher solved precision.

Keywords: green vehicle routing problem, pickup and delivery, tent map, firefly algorithm, time window

1. Introduction

In recent years, global warming and climate change have aroused people's concern. Transportation is one of the most important triggers of global warming and climate change as [Ibrahim et al. \(2020\)](#) proposes. According to the [EPA \(2020\)](#) study, the transportation sector emits the most greenhouse gases from burning fossil fuels in cars, trucks, ships, trains, and airplanes. [Li & Zhang \(2014\)](#) remark that in China, 30 percent of carbon emissions are caused by carbon emissions from transporting goods, and more than 40 percent of a company's total cost is used for transportation fuel consumption. Hence consideration of fuel consumption in logistics will not only help to reduce logistics costs but also lead to fewer carbon emissions.

The classical vehicle routing problem (VRP) has been proposed for more than 60 years. The goal of VRP is to identify a route plan that is served by a fleet of homogeneous vehicles for a group of customers, each customer is visited once by a vehicle, each route starts and ends at the depot and satisfies several side constraints. As a variant of VRP, the vehicle routing problem with pick-up and delivery (VRPPD) is developed by [Nagy & Salhi \(2005\)](#), the problem aims to plan the route of transport vehicles between the pick-up and delivery points. In VRPPD, the vehicle load is no longer monotonously increasing or decreasing. This characteristic complicates routing decisions. In response to an ever-increasing concern about climate change, researchers have extended the VRP to consider environmental impacts. The green vehicle routing problem (GVRP) is conceived by [Erdoğan & Miller-Hooks \(2012\)](#). The objective of the problem is to set low-cost routes within a depot for a homogeneous set of vehicles. Vehicles serve a geographically distributed set of customers within a limited fuel capacity and time.

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The GVRP is characterized by the objective of reconciling the environmental and economic costs. The objective is achieved through the design of effective routes to meet both the ecological and financial concerns under various constraints imposed by different models. For the GVRP, the carrier firms and the customers each have their concerns. The carrier firms focus on reducing carbon emissions when designing vehicle routes. The concern is partly attributable to emissions constraints or carbon policies and partly due to cost-saving considerations. Load factor is one of the essential considerations for vehicle routes as the vehicle load varies when the vehicle is pick-up or delivering, resulting in different emission levels. From the customers' perspective, the product needs to be delivered within the specified time windows. To meet the needs of customers, deliveries outside the time window may result in penalties being paid by the carrier firm.

Recent research on minimizing carbon emissions in models related to the GVRP can be divided into two main categories. In the first category, a load of a vehicle is presumed to be fixed or variations of vehicle load on fuel consumption are ignored. It is modeled by considering the impact of factors such as vehicle speed and customer satisfaction on costs and carbon emissions. One of the shortcomings of the first category models is the lack of load-related assumptions when considering carbon emissions, especially in VRPPD where the load of the vehicle varies constantly, resulting in different emission levels. In the second category, the vehicle load is considered constant within the path and changes as the vehicle shifts from one path to path. The level of vehicle emissions is related to vehicle load.

This paper develops a model for green vehicle routing problems with pick-up and delivery (GVRPPD) by considering changes in vehicle loads and soft time windows for serving customers. The problem is formulated as a mixed integer nonlinear programming (MINLP) model. The proposed model is integrated with a fuel consumption calculation method that considers the load of the vehicle and transportation distance. The type of problem is highly complex and requires considerable computing time for the generation of solutions. An improved firefly algorithm (IFA) with chaos strategy and differential evolution strategy is developed to solve the problem. As compared to the existing GVRP models, our model is new in the sense that it considers the various vehicle loads and the corresponding fuel consumption. To the best of our knowledge, there is no literature on the effect of a variable load of GVRPPD on fuel consumption minimization. The proposed model also considers the penalty evaluation with soft time windows constraint and the capacity constraint in GVRP. The proposed model and the solution based on IFA are verified through numerical experimentation the results show that the proposed model can significantly reduce fuel consumption while ensuring a small change in penalty costs when considering varies in vehicle load. The performance of the proposed IFA is investigated by calculating.

Our contributions are as follows : (1) we introduced GVRPPD and formulated its MINLP model; (2) we propose an efficient and improved firefly algorithm to solve this problem; (3) we present new benchmark results from the literature to compare the performance of IFA. The rest of the paper is organized as follows: Section 2 provides a review of the relevant literature. Section 3 gives the mathematical formula of the problem. Section 4 details the proposed IFA solution. Section 5 presents the computational study and discusses the numerical results. Finally, the conclusion, limitations of the proposed model, and possible future research directions are given in the last section.

2. Literature review

Since the VRP was introduced more than 60 years ago, many models of VRP extensions have been studied and the literature on VRP is very rich. In addition to the traditional VRP, many variants have been developed due to the introduction of practical constraints related to real-world problems, such as the capacitated VRP, the VRP with time windows, the heterogeneous fleet VRP, multiple depots VRP, and the

VRPPD. There is also literature proposing many accurate and heuristic methods for solving VRP variants. This part will elaborate on the existing literature.

Erdoğan & Miller-Hooks (2012) research the GVRP by considering the refueling of alternative fuel vehicles. Küçükoğlu et al. (2015) develop the GVRP integrated with a fuel consumption calculation algorithm and constructed a memory structure adapted simulated annealing metaheuristic algorithm. Considering the fuel level of vehicles, Sadati & Çatay (2021) add gas stations to carry out multi-depot GVRP. Bruglieri et al. (2019) discuss the GVRP with vehicles using alternative fuels. The fuel consumption is assumed to be linearly proportional to the travel distance. They developed a path-based solution approach to solve the problem and prove its effectiveness. Poonthalir & Nadarajan (2018) discuss a bi-objective fuel-efficient GVRP with varying speed constraints. Goal programming is used to minimize both route cost and fuel consumption. Koyuncu & Yavuz (2019) consider three different types of refueling policies to shorten the time for refueling stops. The authors used arc-duplicating and name-duplicating formulations to solve the problem, and their study reveals that, in most cases, arc-duplicating formulation prefers better. Behnamian et al. (2023) take into account situations where the vehicle is running low on clean fuel and thus needs to be refueled. They present a mathematical formulation for a green heterogeneous vehicle routing problem and develop a firefly algorithm to solve the problem in large-size instances. In considering the case of petrol stations, Bruglieri et al. (2022) further introduce the assumption that there are a certain number of petrol pumps at petrol stations. Multiple vehicles are allowed to refuel at the same station, which is more in line with reality. A greedy randomized adaptive search procedure is proposed for the problem and has proven to be effective. Luo et al. (2023) study a time-dependent green vehicle routing problem with the consideration of traffic congestion to formulate logistics plans with the consideration of fuel consumption and carbon emissions reduction. A BP algorithm is designed to handle the time-dependent travel time and time-dependent carbon emissions. Wang et al. (2022) conceive an improved ant colony optimization algorithm to solve the soft time window green vehicle path problem with the heterogeneous fleet. Liu et al. (2023) study the time-dependent green vehicle routing problem with time windows. To reduce total carbon emissions under time-dependent travel times and time window constraints, the problem determines the departure times for vehicles from the depot and the customer nodes along each route. They also propose a method based on a branch-cut-and-price algorithm to solve the problem. However, the exact solution approaches are nevertheless suitable to solve middle-sized instances. Elgharably et al. (2023) develop three multi-objective models, which consider uncertainties in travel time, service time, and demands respectively.

Vehicle fuel costs is related to many factors, such as vehicle load, speed, road slope, and so on. Various models for calculating the fuel costs have been proposed. Scora & Barth (2006), and Barth & Boriboonsomsin (2008) estimate fuel costs based on a comprehensive emissions model. Xiao et al. (2012) propose a fuel cost-oriented model that is more in line with the actual situation of CVRP. Kuo (2010) develop a model for calculating fuel consumption that takes into account speed, time, and vehicle load. Olivera et al. (2015) combine particle swarm optimization with the traffic emission model for the reduction of pollution and fuel consumption in urban areas.

Another variant of the VRP is that the vehicle is restricted by its capacity. Yu et al. (2019) conceive a modified branch and price algorithm to accurately solve the green vehicle routing problem for heterogeneous fleets with Time Windows. Salhi & Nagy (1999) investigate the clustering insertion heuristic to solve the single-station and multi-station VRPPD problems. To obtain the optimal solution for VRPPD, Dell'Amico et al. (2006) take into account column generation, dynamic programming, branching, and pricing and propose an accurate optimization algorithm. Madankumar & Rajendran (2018) propose a model for solving the GVRPPD in a semiconductor supply chain, and the objective is to find the set of minimum-cost routes and schedules for the vehicles. Subramanian et al. (2013) develop the branch cutting and Pricing

Table 1: A summary of the literatures

Literature	Fuel costs	Load	Pick-up and delivery	Time window	Penalty costs
Küçüköğlu et al. (2015)	✓	✓		✓	✓
Poonthalir & Nadarajan (2018)	✓				
Koyuncu & Yavuz (2019)	✓				
Poonthalir et al. (2015)	✓				
Yanik et al. (2014)		✓	✓	✓	
Madankumar & Rajendran (2018)	✓				
Xu et al. (2019)	✓	✓	✓		
Behnamian et al. (2023)	✓	✓		✓	✓
Elgharably et al. (2023)	✓				
Foroutan et al. (2020)	✓			✓	✓
this paper	✓	✓	✓	✓	✓

approach and solve the mixed VPRRD. Xu & Wei (2023) study the dynamic pick-up and delivery problem with transshipments and last-in-first-out constraints. The implementable strategy for managing transportation policies of auto spare part distributors proposed by them can reduce the environmental and economic costs. Yanik et al. (2014) propose a model for the multiple-pickup single-delivery VRP in the presence of multiple vendors and develop a hybrid solution approach. Gao et al. (2023) propose a hybrid ant colony optimization to solve the capacitated vehicle routing problem and the proposed is proven to be competitive with other algorithms. Xu et al. (2019) study the capacitated GVRP with time-varying vehicle speed and soft time. Based on vehicle load, Lin et al. (2014) investigate a variant of VRPPD that aims to minimize Carbon dioxide emissions. Vincent et al. (2021) consider realistic energy consumption, partial recharging policy, and carbon emissions in green mixed fleet vehicle routing problems. An adaptive large neighborhood search heuristic is proposed for solving the problem and its performance is proven. HadjTaieb et al. (2022) expand GVRP in the home healthcare field. They focus on both economic and environmental objectives. A multi-objective green transportation problem for damaging items is proposed by Aktar et al. (2022). Vehicle routing is scheduled by weighing the goals such as minimization of operational cost, carbon emission amount, and so on. Foroutan et al. (2020) study the green vehicle routing and scheduling problem with the heterogeneous fleet in collecting returned goods situation.

VRPPD can be further divided into two subcategories: unpaired and paired. The first category refers to the situation where the pick-up and delivery locations are not matched, and the goods picked up at any pick-up location can be used to meet the needs of any delivery customer according to Rieck et al. (2014). The second category, known as the one-to-one pick-up and Delivery problem (OPDP), involves transportation requests, each associated with a starting point and a destination, resulting in paired pick-up and delivery points. OPDP considers shipping requests, each associated with a starting point and a destination and resulting in paired pick-up and delivery points proposed by Chen et al. (2015). The problem studied in this paper belongs to the first category. As far as we know, the current studies on GVRPPD. Mulvey et al. (1995) take into account the economic costs and environmental costs. The problem solved by Madankumar & Rajendran (2018) belongs to the one-to-one pick-up and delivery problem. Therefore, to reduce fuel emissions and improve customer satisfaction, this paper establishes a green pick-up and delivery vehicle path planning problem with a time window and designs an improved algorithm to solve the problem.

3. Model formulation

In logistics, in the courier industry like JD, SF, and so on, vehicles provide services to customers that include not only delivery services but also pick-up services. In our proposed model, the vehicle provides the customer with goods shipped from the supplier and picks up the goods that need to be sent. All goods are assumed to be homogeneous to simplify the model. The problem is described as follows: there are a set of requests and a set of vehicles, each request requires the goods to be transported to a delivery vertex or solicited from a pick-up vertex. The vehicle routing is planned according to the residual fuel level of vehicles, loads of vehicles, and time windows for serving customers. In addition, one of the main parts of the objective function is to reduce fuel consumption. This section first state the definition of the problem in Section 3.1. And then, in Section 3.2, the fuel consumption function based on the load of the vehicle is addressed. Finally, the model of GVRPPD is described as a single objective optimization in Section 3.3.

3.1. Problem definition

The problem is defined on a complete graph $G = \{V, E\}$, where the vertex set V is a combination of the depot set $\{v_0\}$ and the customer's location set $V_c = \{v_1, v_2, \dots, v_n\}$. e_{ij} in the set $E = \{e_{ij}\}$ corresponds to the edge connected by vertex v_i and vertex v_j . Each edge e_{ij} has measures of traveling time t_{ij} and distance d_{ij} . Each customer has a pick-up or delivery request and each customer has a time window.

A group of homogeneous vehicles $V_h = \{vh_1, vh_2, \dots, vh_m\}$ provides service to customers with constant travel speed. The vehicles provide service to customers within the time windows. If the time windows are violated, penalty costs will be incurred accordingly. y_{ki} means the residual fuel level of vehicle vh_k when the vehicle arrives at the vertex i . When the vehicle starts from the depot, the initial fuel level equals Y . The goal of the problem is to reduce fuel costs and penal costs for violating time windows.

Vehicle routes should meet the following restrictions:

- (1) Each customer's pick-up or delivery demand must be met by one vehicle.
- (2) The total load must not exceed the vehicle's capacity at any vertex on the route.
- (3) Each route starts and ends at the depot.
- (4) The number of vehicles used does not exceed the maximum number of available vehicles.
- (5) The fuel consumption of each route must not exceed the fuel level of the vehicle.

To make it easier for readers to understand these notations, a simple example of a route plan is presented in Figure 1. In Figure 1, there are a total of 15 vertices including 1 depot and 14 customers. v_0 denotes the depot, and v_1 to v_{14} represent customers. There are total 3 paths, respectively $v_0 \rightarrow v_7 \rightarrow v_{12} \rightarrow v_3 \rightarrow v_4 \rightarrow v_0$, $v_0 \rightarrow v_{10} \rightarrow v_9 \rightarrow v_{14} \rightarrow v_8 \rightarrow v_{13} \rightarrow v_0$, and $v_0 \rightarrow v_6 \rightarrow v_{11} \rightarrow v_1 \rightarrow v_2 \rightarrow v_0$. The arrow between two vertices represents a directed arc. For example, (v_7, v_{12}) represents the arc of $v_7 \rightarrow v_{12}$.

3.2. Calculation of fuel consumption

Fuel consumption depends not only on the distance traveled, but is also related to many factors. In this paper, the fuel consumption rate function proposed by Xiao et al. (2012) is used, where fuel consumption is described using vehicle load and traveled distance. In this function, The fuel consumption rate is proportional to the distance traveled and the vehicle load, which can be described as Eq. (1),

$$p(Q_1) = \sigma(Q_0 + Q_1) + b. \quad (1)$$

Q_0 and Q_1 mean the weight of the vehicle and the vehicle load, respectively. b refers to a constant related to the fuel consumption of the vehicle. Therefore, the fuel consumption rate of empty and full vehicles can be expressed as Eq. (2) and Eq. (3).

$$\rho_0 = \sigma Q_0 + b \quad (2)$$

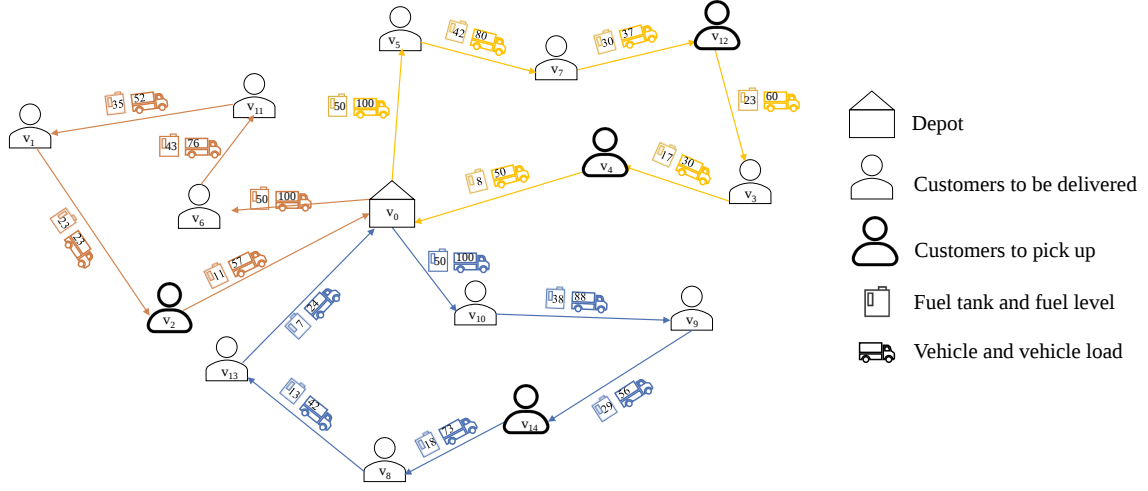


Figure 1: An example of vehicle routes.

$$\rho^* = \sigma(Q_0 + Q) + b \quad (3)$$

According to Eq. (2) and Eq. (3), σ is expressed as Eq. (4),

$$\sigma = (\rho^* - \rho_0)/Q \quad (4)$$

Therefore, the fuel consumption rate for a vehicle load of Q_1 is calculated as Eq. (5),

$$p(Q_1) = \rho_0 + \left(\frac{\rho^* - \rho_0}{Q}\right)Q_1 = \rho_0 + \sigma Q_1 \quad (5)$$

3.3. Mathematical model

To describe the model of GVRPPD, the following notations are used and summarized in Table 2.

The model uses the following three decision variables,

$$x_{ijk} = \begin{cases} 1 & \text{vehicle } v_k \text{ goes from vertex } v_i \text{ to } v_j \\ 0 & \text{otherwise} \end{cases}$$

U_{ij} : The delivery load carried by vehicles on the edge e_{ij}

V_{ij} : The pick-up load carried by vehicles on the edge e_{ij}

In this paper, the effect of load on vehicle fuel costs is considered. The objective function for minimizing the fuel costs by considering the vehicle load and traveling distance is given as Eq. (6),

$$\min J_1 = \min \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^m p_f \cdot d_{ij} x_{ijk} (\rho_0 + \sigma(U_{ij} + V_{ij})) \quad (6)$$

The other objective of this paper is to minimize penalty costs for violating time windows. Different from the hard time windows, the soft time windows can be violated and lead to violation as Eq. (7) and Eq. (8). The objective function is defined as Eq. (9),

$$\max\{0, t_{ik} - a_i\} = \alpha_i, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (7)$$

$$\max\{0, a_i - t_{ik}\} = \beta_i, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (8)$$

Table 2: Meaning of the notations

Notation	Meaning
v_0	The depot
V_c	The customer's location set
V	$V = \{v_0\} \cup V_c$
d_{ij}	Traveled distance between v_i and v_j
t_{ij}	Traveled time between v_i and v_j
t_{ik}	Time of vehicle arriving at v_i
st_i	The time the vehicle provides service at v_i
$[a_i, b_i]$	The time window of v_i
λ	Penalty coefficient for violating time windows
D_i	Delivery demand of v_i
P_i	pick-up demand of v_i
Q	Vehicle capacity
V_h	The set of vehicles
p_f	Oil price per gallon
Y	Initial fuel level

$$\min J_2 = \sum_{i=1}^n \lambda(\alpha_i + \beta_i) \quad (9)$$

The objective function of the GVRPPD is to minimize the total costs given as Eq. (10),

$$\min J = J_1 + J_2 = \sum_{i=0}^n \sum_{j=0}^n \sum_{k=1}^m p_f \cdot d_{ij} x_{ijk} (\rho_0 + \sigma(U_{ij} + V_{ij})) + \sum_{i=1}^n \lambda(\alpha_i + \beta_i) \quad (10)$$

Since this paper investigates GVRPPD extended from VRP, the vehicle routing is also subject to the residual fuel level and the load in addition to general constraints as Eq. (11) - (23).

$$\sum_{i=0}^n \sum_{k=1}^m x_{ijk} = 1, \quad j = 1, 2, \dots, n \quad (11)$$

$$\sum_{i=0}^n x_{ijk} - \sum_{i=0}^n x_{jik} = 0, \quad j = 1, 2, \dots, n, \quad k = 1, 2, \dots, m \quad (12)$$

$$\sum_{j=1}^n \sum_{k=1}^m x_{0jk} \leq m, \quad (13)$$

$$\sum_{i=1}^n \sum_{k=1}^m x_{i0k} \leq m, \quad (14)$$

$$\sum_{i=0}^n U_{ij} - \sum_{i=0}^n U_{ji} = D_j, \quad j = 1, 2, \dots, n \quad (15)$$

$$\sum_{i=0}^n V_{ji} - \sum_{i=0}^n V_{ij} = P_j, \quad j = 1, 2, \dots, n \quad (16)$$

$$U_{ij} + V_{ij} \leq Q \cdot \sum_{k=1}^m x_{ijk}, \quad i = 0, 1, \dots, n, \quad j = 0, 1, \dots, n \quad (17)$$

$$y_{jk} \leq y_i - [\rho_0 + \sigma(U_{ij} + V_{ij})] d_{ij} x_{ijk} + Y(1 - x_{ijk}), \quad i = 0, \dots, n, \quad j = 0, \dots, n, \quad k = 1, \dots, m \quad (18)$$

$$y_{ik} \geq (\rho_0 + \sigma(U_{ij} + V_{ij})) d_{ij} + (\rho_0 + \sigma(U_{ij} + V_{ij})) d_{j0}, \quad i = 1, 2, \dots, n \quad (19)$$

$$y_{ik} \geq (\rho_0 + \sigma(U_{ij} + V_{ij})) d_{i0}, \quad i = 1, 2, \dots, n \quad (20)$$

$$(t_{ik} + st_i + t_{ij}) \leq t_{jk} + M(1 - x_{ijk}), \quad i = 0, 1, \dots, n, \quad j = 0, 1, \dots, n, \quad k = 1, 2, \dots, m \quad (21)$$

$$x_{ijk} \in \{0, 1\} \quad i = 0, 1, \dots, n, \quad j = 0, 1, \dots, n, \quad k = 1, 2, \dots, m \quad (22)$$

$$U_{ij}, V_{ij} \geq 0, \quad i = 0, 1, \dots, n, \quad j = 0, 1, \dots, n \quad (23)$$

Constraint (11) guarantees that every vertex will be serviced by one and only one vehicle. Constraint (12) guarantees that any vehicle will leave the vertex where it arrives, thereby ensuring that the vehicle must return to the depot. Constraints (13) and (14) mean the number of vehicles leaving and returning to the depot cannot exceed the number of vehicles owned. Constraint (15) and constraint (16) ensure the pick-up demand or delivery demand of every vertex is met. Constraint (17) ensures that the load on any edge does not exceed the vehicle's capacity. When the vehicle v_{h_k} drives from vertex v_i to v_j , the residual fuel level will decrease, and the change of the residual fuel level of the vehicle on the e_{ij} is described through constraint (18). Constraint (19) and constraint (20) ensure that when the vehicle is scheduled to travel from v_i to v_j , there is sufficient residual fuel level to return to the depot after serving v_j or directly to the depot so that the vehicle does not stop halfway due to insufficient fuel. Constraint (21) is used to describe the time of vehicle vh_k arrival at v_j . Constraint (22) and Constraint (23) specify the value range of decision variables.

4. Solution method

Yang (2009) proposes FA based on the luminous behavior of fireflies in nature. As an emerging swarm intelligence optimization algorithm, the FA has the advantage of fewer parameters and is easier to understand. To simulate the phenomenon of real fireflies glowing, Yang (2009) proposes three rules: 1) all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of its sex; 2) the brightness of firefly determines the size of its attraction, and the brighter firefly attract darker firefly, the considered firefly will move randomly if no firefly is brighter than it; 3) the optimal value of the function is proportional to the brightness of the firefly.

The algorithm has not been proposed for a long time and the improvement and optimization of FA are not yet complete. The firefly algorithm has the disadvantage of slow convergence speed and has a high possibility of being trapped in a local optimum. The differential evolution (DE) algorithm proposed by Storn & Price (1997) overcomes the main disadvantage of lack of global search ability and robustness. DE is a stochastic population-based search technique that uses three evolutionary operators: mutation, crossover, and selection, to guide the search to find optimal solutions. It is considered a powerful tool for solving optimization problems in continuous spaces. In contrast, GVRPPD is an NP-hard combinatorial problem that must be addressed by a discrete search space. Therefore, the solutions found by DE are continuous vectors that must be converted to vehicle routing. This study offered an improved firefly algorithm (IFA) based on the advantages of FA and DE to solve the GVRPPD. This section proposes five main steps on IFA, such as (1) convert the solution to vehicle routing, (2) initialize the population of fireflies whose positions refer to the solutions, (3) calculate the brightness of fireflies, (4) calculate attractiveness, (5) update the position, (6) improve candidate solutions by DE.

4.1. Converting solution to vehicle routing

In this step, a solution generated randomly is converted to vehicle routing by the best-matched value (BMV) mapping method. According to Ali et al. (2019), BMV is an appropriate procedure for transforming continuous numbers into discrete numbers. This procedure is popular and easy to use for combinatorial optimization problems. The main process of BMV is as follows: (1) the components of the continuous vector are ranked from the smallest to largest; (2) the smallest in the vector is replaced by its ranked discrete index to produce an integer vector. Experimental results demonstrate that this method outperforms the other mapping methods. The following example of a solution vector $\mathbf{x}_i$ to a problem with six customers illustrates the steps of BMV. In Figure 2, the first row shows the customers, the second row shows the components $\mathbf{x}_i$ of $\mathbf{x}_i$ and the third row shows the vehicle routing.

customers	1	2	3	4	5	6
x_{ij}	0.52	0.46	0.13	0.89	0.26	0.37
vehicle routing	5	4	1	6	2	3

Figure 2: Converting of a solution to vehicle routing

For GVRPPD, there is more than one vehicle and the vehicles are constrained by loads and fuel levels. BMV is adjusted to convert the solutions. The following example of a solution vector $\mathbf{x}_i$ to GVRPPD with six customers and two vehicles illustrates the adjusted BMV. In Figure 3, $h_{ik,1}$ and $h_{ik,2}$ in the matrix $\mathbf{H}_1$ and $\mathbf{H}_2$ mean the traveling time and the time window violation when vehicle vh_k travels to vertex i from the current vertex. In Figure 3 the current vertex refers to the depot. $h_{ik,3}$ in the matrix $\mathbf{H}_3$ is the components of the solution vector $\mathbf{x}_i = [0.15, 0.35, 0.56, 0.32, 0.53, 0.49]$. If the fuel level or load exceeds the limit when the vehicle vh_k travels to vertex v_i , $h_{ik,3}$ will equal $+\infty$. When converting the solution vector $\mathbf{x}_i$ to the vehicle routing, we not only rank the components of the solution vector but also consider the traveling time and the time window violation. The upper limit ub and lower limit lb of components of the solution vector equals 1 and 0, respectively. h_{ik} in matrix $\mathbf{H}$ is calculated through Eq (24)-(26). When $h'_{ik} = \min_{1 \leq i \leq n} \min_{1 \leq k \leq m} h_{ik}$ is found, v_i is added to the vehicle routing of vh_k and removed from the list. Then, $\mathbf{H}$ is updated and the next vertex is added until all vertices are removed. Vehicle routing is converted through matrix $\mathbf{H}$ as Figure 4.

$$h1'_{ik} = \frac{(h1_{ik} - \min_{1 \leq i \leq n} \min_{1 \leq k \leq m} h1_{ik})}{\max_{1 \leq i \leq n} \max_{1 \leq k \leq m} h1_{ik} - \min_{1 \leq i \leq n} \min_{1 \leq k \leq m} h1_{ik}} \quad (24)$$

$$h2'_{ik} = \frac{(h2_{ik} - \min_{1 \leq i \leq n} \min_{1 \leq k \leq m} h2_{ik})}{\max_{1 \leq i \leq n} \max_{1 \leq k \leq m} h2_{ik} - \min_{1 \leq i \leq n} \min_{1 \leq k \leq m} h2_{ik}} \quad (25)$$

$$h_{ik} = h1'_{ik} + h2'_{ik} + h3_{ik} \quad (26)$$

vh_1	vh_2	i	vh_1	vh_2	i	vh_1	vh_2	i
0.47	0.47	1	14.73	14.73	1	0.15	0.15	1
0.52	0.52	2	13.23	13.23	2	0.35	0.35	2
0.40	0.40	3	0.68	0.68	3	0.56	0.56	3
0.45	0.45	4	11.66	11.66	4	0.32	0.32	4
0.38	0.38	5	0	0	5	0.53	0.53	5
0.48	0.48	6	9.88	9.88	6	0.49	0.49	6

Figure 3: Best-matched value of the solution

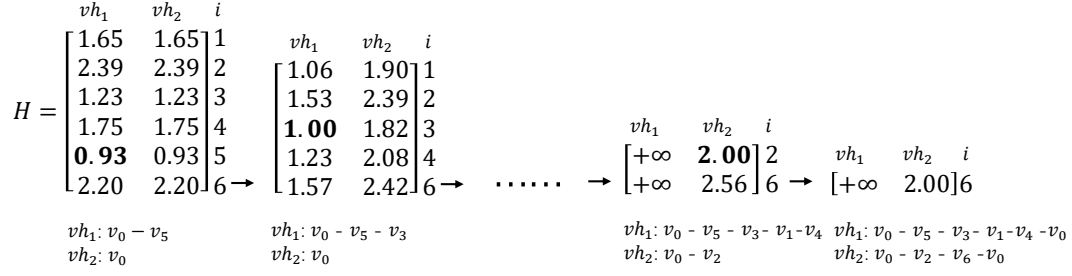


Figure 4: The illustration of solution converting

4.2. Population initialization

Chaos theory was established in the 1970s. Chaos is unpredictable and aperiodic which can be used to improve the optimization efficiency of the algorithm proposed by Kaur et al. (2020). As given by Ali et al. (2019), the basic idea of chaotic mapping is to map the optimization variable linearly to the chaos variable, then carry out the optimization search according to the ergodicity and randomness of the chaos, and finally convert the obtained solution linearly to the optimization variable space. Tent map is a linear chaotic mapping that presents good pseudo-randomness. The optimal search based on these characteristics can effectively maintain the diversity of the population and significantly improve global search ability according to Li et al. (2020). This paper uses the tent map to construct the initial population. The function of the tent mapping is as Eq. (27),

$$\eta'_j = \begin{cases} \eta_j/\varphi & \eta_j \in [0, \varphi) \\ (1 - \eta_j)/(1 - \varphi) & \eta_j \in (\varphi, 1] \end{cases} \quad (27)$$

φ is the control parameter. This paper adds tent mapping to maintain the diversity of the initial population as Algorithm 1. The solution vector $\mathbf{x}_i$ of the initial population is generated as Eq (28), x_{ij} is the j th component of $\mathbf{x}_i$. The chaotic vector $\tilde{\mathbf{x}}_i$ of $\mathbf{x}_i$ shows as Eq (29). $f(\mathbf{x}_i)$ means the value of the objective function (10) according to the vehicle routing converted from solution vector $f(\mathbf{x}_i)$,

$$x_{ij} = rand_{ij}(ub - lb) + lb, \quad j = 1, 2, \dots, n \quad (28)$$

$$\tilde{x}_{ij} = \eta'_j(ub - lb) + lb, \quad j = 1, 2, \dots, n \quad (29)$$

$rand_{ij}$ randomly distributes in $[0, 1]$, $\tilde{\mathbf{x}}_i$ is the j th component of $\tilde{\mathbf{x}}_i$.

For each solution vector $\mathbf{x}_i$, the corresponding chaotic vector $\tilde{\mathbf{x}}_i$ is generated by the tent mapping. Then, the vector with a better value of the objective function is selected to form the initial population.

4.3. Brightness

The distance between two fireflies is calculated according to Eq (30),

$$r_{il} = \|\mathbf{x}_i - \mathbf{x}_l\| = \sqrt{\sum_{k=1}^n (x_{ik} - x_{lk})^2} \quad (30)$$

where solution vectors $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{i(n)})^\top$ and $\mathbf{x}_l = (x_{l1}, x_{l2}, \dots, x_{ln})^\top$ are n -dimensional vectors which mean the location of firefly ff_i and ff_l . x_{ik} and x_{lk} mean the k th component of $\mathbf{x}_i$ and $\mathbf{x}_l$. Eq (30) calculates the distance between two fireflies by accumulating the difference of the components of the solution vectors.

The brightness of a firefly at position $\mathbf{x}_i$ can be expressed as $I(\mathbf{x}_i) \propto f(\mathbf{x}_i)$. Distance r_{ij} between fireflies ff_i and ff_j varies. Light intensity decreases with the distance from its source and light is also absorbed in

Algorithm 1 Chaotic initialization algorithm for population

Require: The customer's set V_c , the depot set $\{v_0\}$, φ , and population size pop_size

Ensure: The initial population after tent mapping

1. Generates $rand_{ij}$ through random distribution
 2. Generate initial solution $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{in})^\top, i = 1, 2, \dots, pop_size$ through Eq (28)
 3. Generates $\eta_j, j = 1, 2, \dots, n$ through random distribution
 4. **for** $i = 1 : pop_size$ **do**
 5. **for** $j = 1 : n$ **do**
 6. Generates the component $\tilde{x}_{ij}$ of $\tilde{\mathbf{x}}_i = (\tilde{x}_{i1}, \tilde{x}_{i2}, \dots, \tilde{x}_{in})^\top$ through Eq (29)
 7. Update η_j through Eq (27)
 8. $\eta_j \leftarrow \eta'_j$
 9. **end for**
 10. **if** $f(\tilde{\mathbf{x}}_i) < f(\mathbf{x}_i)$ **then**
 11. $\mathbf{x}_i \leftarrow \tilde{\mathbf{x}}_i$
 12. **end if**
 13. **end for**
-

the media. Considering a media with a fixed light absorption coefficient γ and the distance r_{il} between ff_i and ff_l , the light intensity of ff_i at the position of ff_l can be described as Eq (31),

$$I(r_{il}) = I(\mathbf{x}_i)e^{-\gamma r_{il}^2} \quad (31)$$

$I(\mathbf{x}_i)$ means the brightness of ff_i .

Since the GVRPPD is about a minimization problem, the brightness of fireflies is negatively correlated with the value of the objective function (10). Function (32) describes the light intensity of ff_i at the position of ff_l ,

$$I(r_{il}) = \frac{e^{-\gamma r_{il}^2}}{f(\mathbf{x}_i)} \quad (32)$$

Through Eq (32), the brightness of a firefly is related to the economic cost of the solution it represents.

4.4. Attractiveness

The attractiveness of a firefly is considered to be related to its brightness. The attractiveness $\beta(r_{il})$ of firefly ff_i for firefly ff_l can be defined as Eq (33), where β_0 is the attractiveness at $r_{il} = 0$,

$$\beta(r_{il}) = \beta_0 e^{-\gamma r_{il}^2} \quad (33)$$

If firefly ff_i is brighter, then ff_l is less attractive to ff_i . Therefore, the attractiveness of firefly ff_i to ff_l is inversely proportional to the brightness of ff_j , which can be described as Eq (34),

$$\beta(r_{il}) = \beta_0 e^{-(\gamma r_{il}^2)I(r_{il})/I(r_{ii})} \quad (34)$$

$I(x_l)$ means the brightness of firefly ff_l . Eq (34) introduces the brightness of fireflies into the measure of attractiveness. This means that firefly considers more factors in updating location.

The brightness and attractiveness can be calculated according to formulas (32) and (34). Then we can find the brightest firefly in the current population, which means the current optimal solution.

4.5. Position updating

When firefly ff_i is attracted to the brighter one ff_l the location of firefly ff_i will be updated by Eq (35),

$$\mathbf{x}_i = \mathbf{x}_i + \beta(r_{il})(\mathbf{x}_l - \mathbf{x}_i) + \alpha(rand - \frac{1}{2}) \quad (35)$$

$\beta(r_{il})(\mathbf{x}_l - \mathbf{x}_i)$ describes position updation due to the attractiveness of ff_l to ff_i . $\alpha \in [0, 1]$ is the fixed randomization parameter and $rand$ is a random number generator uniformly distributed in $[0, 1]$. If ff_i is the current brightest firefly then it will move randomly according to $\alpha(rand - \frac{1}{2})$.

In the standard firefly algorithm, α is fixed throughout all iterations. This will result in the algorithm easily getting trapped in the local optima and causing low precision as Yu et al. (2015) describe. To remedy this defect, the factor α is adjusted adaptively through Eq (36),

$$\alpha = 1 - \frac{iter}{\max_iter} \quad (36)$$

$iter$ refers to the current number of iterations while $\max_iter$ is the maximum number of iterations. According to Yu et al. (2015), the adaptability of α decreases as the number of iterations increases which makes firefly focus more on global exploration at first and on local exploration later.

4.6. Differential evolution

The steps of DE can be described as follows,

Mutation: Three different solution vectors are randomly sampled from the population to generate variation vector $\mathbf{v}_i$ through Eq (37),

$$\mathbf{v}_i = \mathbf{x}_{r_1} + F(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}) \quad (37)$$

where $\mathbf{v}_i$ means mutation vector of $\mathbf{x}_i$. $\mathbf{x}_{r_2} - \mathbf{x}_{r_3}$ is a difference vector, F is the scaling factor which generally takes the value of $[0, 2]$, and $i \neq r_1 \neq r_2 \neq r_3$.

$\mathbf{x}_{best}$ is used to represent the current optimal solution. Then other solutions in the population can obtain variation vectors through $\mathbf{x}_{best}$. This process is given as Eq (38),

$$\mathbf{v}_i = \mathbf{x}_{best} + F(\mathbf{x}_{r_2} - \mathbf{x}_{r_3}) \quad (38)$$

Crossover: Individual $\mathbf{x}_i$ in gth are cross-manipulated according to Eq (39),

$$u_{ij} = \begin{cases} v_{ij} & \text{if } cr(j) \leq CR \text{ or } j = rand_n \\ x_{ij} & \text{else} \end{cases} \quad (39)$$

where u_{ij} means the j th component of the current crossover vector $\mathbf{u}_i$ of individual $\mathbf{x}_i$. v_{ij} and x_{ij} refer to the j th component of the current mutation vector $\mathbf{v}_i$ and $\mathbf{x}_i$, respectively. $cr(j) \in [0, 1]$ is the j th evaluation of a uniform random number generator and CR selected randomly in $[0, 1]$ is the crossover constant. $rand_j$ is chosen at random with equal probability from $[1, 2, \dots, n]$ which ensures that at least one component of the crossover vector is produced by the mutant manipulation. This process is shown in Figure 5.

Selection: DE uses the greedy criterion to select individuals to move into the next generation according to Eq (40).

$$\mathbf{x}_i = \begin{cases} \mathbf{u}_i & \text{if } f(\mathbf{u}_i) \leq f(\mathbf{x}_i) \\ \mathbf{x}_i & \text{else} \end{cases} \quad (40)$$

Based on the above-improved operations, the pseudocode of the firefly algorithm based on chaotic mapping and differential evolution proposed in this paper is given in Algorithm 2.

Algorithm 2 The improved algorithm for model

Require: The customer's set V_c , the depot set $\{v_0\}$

Ensure: x_g

1. Initialize parameters: $\gamma, \beta_0, max_iter, pop_size, \varphi$
 2. Generate $rand_{ij}$ through random distribution
 3. Generate initial solution $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^\top, i = 1, 2, \dots, pop_size$ through Eq (28)
 4. Generate $\eta_j, j = 1, 2, \dots, n$ through random distribution
 5. **for** $i = 1 : pop_size$ **do**
 6. **for** $j = 1 : n$ **do**
 7. Generate chaotic vector $\tilde{x}_i = (\tilde{x}_{i1}, \tilde{x}_{i2}, \dots, \tilde{x}_{i(n+1)})^\top$ of x_i through Eq (29)
 8. Update η_j through Eq (27)
 9. $\eta_j \leftarrow \eta'_j$
 10. **end for**
 11. **if** $f(\tilde{x}_i) < f(x_i)$ **then**
 12. $x_i \leftarrow \tilde{x}_i$
 13. **end if**
 14. **end for**
 15. **for** $g = 1 : max_iter$ **do**
 16. Update α through Eq (36)
 17. **for** $i = 1 : pop_size$ **do**
 18. Select $f(x_j) < f(x_i)$ at random
 19. Update position of x_i through Eq (35)
 20. **for** $j = 1 : pop_size$ **do**
 21. **if** $f(x_i) < f(x_j)$ **then**
 22. $x_g \leftarrow x_i$
 23. **end if**
 24. **end for**
 25. **end for**
 26. **for** $i = 1 : pop_size$ **do**
 27. Get the crossover vector u_i of x_i by Eq (39)
 28. **if** $f(u_i) < f(x_i)$ **then**
 29. $x_i \leftarrow u_i$
 30. **end if**
 31. **for** $j = 1 : pop_size$ **do**
 32. **if** $f(x_i) < f(x_j)$ **then**
 33. $x_g \leftarrow x_i$
 34. **end if**
 35. **end for**
 36. **end for**
 37. **end for**
-

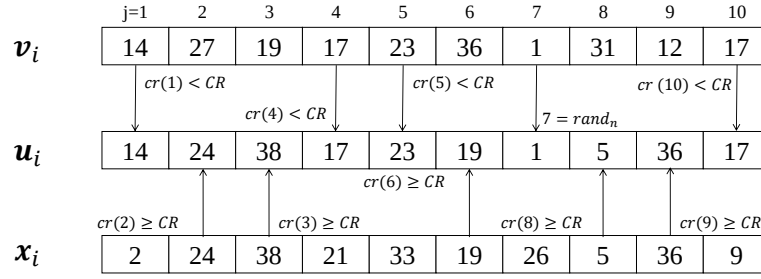


Figure 5: Illustration of crossover

In Algorithm 2, lines 5 to line 13 maintain the diversity of the initial population by tent map. Line 15 is the dynamic adjusting scheme of α . Location updation of fireflies is given in lines 17 and 18. Then, the brightest firefly in the population is found and recorded as x_g in line 21. Line 25 to line 27 get a crossover. vector of other solution vectors and update solution vectors by greedy criterion. Line 29 and line 30 show the updation of x_g . Finally, the optimal vehicle routing is converted from x_g .

5. Numerical experiment

We use problem instances that Li & Lim (2003) generated on the Solomon dataset. Corresponding to the classification of Solomon, Li & Lim (2003) divide the data sets into six categories: *LC1*, *LC2*, *LR1*, *LR2*, *LRC1*, and *LRC2*. Each data set has a central depot, time window constraint, pick-up demand, and delivery demand. According to the law of distribution, the instance can be divided into three categories: *LC*, *LR*, and *LRC*. The customers of instance *LC* are clustered, while the customers of instance *LR* are scattered. Half of the customers of instance *LRC* are clustered and half of the customers are scattered. It is available for download at <https://www.sintef.no/projectweb/top/pdptw/li-lim-benchmark/>.

All experiments were carried out on a computer equipped with Intel(R) Core(TM) i7-6500U CPU @ 2.50GHz and 8 GB RAM. The algorithm was coded in MatlabR2021a. Some of the parameters mentioned in the model are taken as follows according to Fan et al. (2021), Poonthalir & Nadarajan (2018), and Xiao et al. (2012). Parameter Settings are in Table 3.

Table 3: Parameter Setting

Parameter	Meaning	Value
β_0	Maximum attractiveness	1
γ	Light absorption coefficient	1
pop_size	Number of fireflies	20
max_iter	Maximum number of generation	50
F	Scaling factor	[0, 1]
λ	Penalty coefficient for unit time violations of the time window	3
Y	Initial fuel amount	60
p_f	Oil price per gallon	4.22
ρ_0	FCR of the empty vehicle	0.264
ρ^*	FCR of the full vehicle	0.528
φ	The control parameter	0.5

5.1. Carbon emissions

To compare the difference in fuel costs in the two cases with or without considering fuel costs in the objective function, two models are compared. The first model refers to the GVRPPD proposed in 3.3, while the second model has the same constraints as GVRPPD but aims to minimize the penalty costs. The results are reported in Table 4.

In Table 4, columns “fc” indicate the fuel costS obtained by solving the GVRPPD and the compared model, while columns “pc” indicate the penalty costs for violating time windows. “Gap(%)” indicates percentage deviations, where Gap_1 represents the percentage deviation of fuel costs of the two models and Gap_2 represents the percentage deviation of penalty costs of the two models.

When fuel costs are considered in the objective function, the fuel costs decreases significantly compared to not considering fuel costs. Table 4 reveals that the fuel costs is reduced by up to 59.01% and at least 33.75%, with an average reduction of 44.16%. Results show that the green objective has a significant effect on total fuel consumption costs.

It’s been revealed that considering the fuel costs as the objective will lead to an increase in penalty costs. But the increase is small with customer satisfaction dropping by 16.86% at the most, 4.10% at the least, and 9.69% on average. The above results indicate that the GVRPPD can significantly reduce the fuel

costs while increasing the penalty costs. Fuel consumption of logistics vehicles is one of the main sources of carbon emissions. In recent years, fuel emission from logistics vehicles has been a topic of concern for many researchers. This paper presents a GVRPPD model and develops solutions to help enterprises develop green path plans. The model minimizes carbon economic costs while maximizing customer satisfaction by considering soft time windows constraints, customer satisfaction, the fuel consumption function, and vehicle load. We also improved the firefly algorithm to enhance the adoption of this model. A series of computational tests are carried out on different datasets to verify the effectiveness of the model and algorithm. The performance of the algorithm is verified by comparing various algorithms.

This study has three contributions. Firstly, this study establishes a GVRPPD model and evaluates the trade-off between customer satisfaction and fuel consumption in GVRP considering vehicle load. Secondly, by introducing chaotic mapping and differential evolution to deal with GVRPPD, the firefly algorithm is improved. Third, through the statistical analysis of the design experiment, it is found that GVRPPD can significantly reduce carbon emissions without significantly reducing customer satisfaction, which provides an opportunity for logistics enterprises to develop green operations.

Future research may be extended to the mixed fleet of various vehicles with different parameters. Further research may also consider extending the model to the dynamic nature of the business. In this case, dynamically arriving orders can be added to the routing decision dynamically. Fuel consumption of logistics vehicles is one of the main sources of carbon emissions. In recent years, fuel emission from logistics vehicles has been a topic of concern for many researchers. This paper presents a GVRPPD model and develops solutions to help enterprises develop green path plans. The model minimizes carbon economic costs while maximizing customer satisfaction by considering soft time windows constraints, customer satisfaction, the fuel consumption function, and vehicle load. We also improved the firefly algorithm to enhance the adoption of this model. A series of computational tests are carried out on different datasets to verify the effectiveness of the model and algorithm. The performance of the algorithm is verified by comparing various algorithms.

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Future research may be extended to the mixed fleet of various vehicles with different parameters. Further research may also consider extending the model to the dynamic nature of the business. In this case, dynamically arriving orders can be added to the routing decision dynamically. a little. Not only does the GVRPPD result in lower fuel costs, but also reduces the total costs of distribution while paying a low penalty costs.

5.2. The performance of IFA

To test the performance of IFA, we solved the instances using GA, PSO, FA, and IFA. The results are reported in Table 5-7.

In tables, the column “IFA” indicates the improved firefly algorithm proposed by this paper. In other columns, “GA”, “PSO”, and “FA” refer to Genetic Algorithm, Particle Swarm Optimization, and Firefly Algorithm. The columns *best* under each algorithm mean the best solutions obtained by algorithms and the columns *Gap(%)* refer to percentage deviations of IFA from other algorithms. Since the GVRPPD sets penalty costs as the objective, the time window is soft and the service can be timed out or advanced, so the customers in the instance will not be missed due to time constraints.

Results show that compared with GA, PSO, and FA, IFA can get a better solution in all instances. The optimal performance of instance *LC* outperforms GA, PSO, and FA by 36.68%, 17.02%, and 28.23%, respectively. The overall average performance of instance *LC* outperforms GA, PSO, and FA by 31.07%, 14.04%, and 23.74%, respectively. The optimal performance of instance *LR* outperforms GA, PSO, and FA by 21.22%, 13.11%, and 21.28%, respectively. The overall average performance of instance *LR* outperforms GA, PSO, and FA by 18.29%, 9.01%, and 18.06%, respectively. The optimal performance of instance *LRC* outperforms GA, PSO, and FA by 24.23%, 12.97%, and 23.83%, respectively. The overall average performance of instance *LRC* outperforms GA, PSO, and FA by 19.74%, 10.87%, and 19.46%, respectively. IFA performs better on *LC*.

Figures 6-9 show the percentage deviation of the optimal solution obtained by the algorithms from the initial target value for the number of iterations 10, 20, 30, and 40, respectively. It can be observed that in

Table 4: Comparison of the two model

Istance	GVRPPD		the compared model		Gap(%)	
	fc	pc	fc	pc	Gap_1	Gap_2
LC101	2474.50	2263.64	5965.20	2170.88	58.52	4.10
LC102	2916.80	1595.28	5705.20	1524.12	48.87	4.46
LC103	2876.20	1090.04	5585.90	1030.29	48.51	5.48
LC104	2779.5	514.31	4899.80	480.57	43.27	6.56
LC105	2401.00	2143.01	5857.70	2043.35	59.01	4.65
LC106	2337.40	2059.14	5910.80	1960.69	60.46	4.78
LC107	2527.30	1987.29	6154.40	1890.77	58.94	4.86
LC108	2468.90	1858.04	6023.00	1756.03	59.01	5.49
LC109	2678.00	1541.00	5717.50	1459.09	53.16	5.32
LR101	3628.90	434.75	5546.70	385.05	34.58	11.43
LR102	3571.80	348.04	5612.40	304.69	36.36	12.45
LR103	3274.00	220.39	5226.60	188.99	37.36	14.25
LR104	3006.70	109.60	4589.40	91.12	34.49	16.86
LR105	3567.00	391.52	5384.00	342.45	33.75	12.53
LR106	3203.00	304.23	5466.80	260.48	41.41	14.38
LR107	3086.90	223.00	5423.89	197.49	43.09	11.44
LR108	2887.30	133.03	5321.56	119.26	45.74	10.35
LR109	3267.40	324.96	5745.36	290.45	43.13	10.62
LRC101	4006.90	411.96	6580.50	348.10	39.11	15.50
LRC102	3951.20	316.95	6256.00	266.61	36.84	15.88
LRC103	3809.50	207.52	6256.00	266.31	36.84	15.88
LRC104	3376.30	126.31	5984.65	116.86	43.58	7.49
LRC105	3925.20	379.20	6391.50	325.68	38.59	14.11
LRC106	3522.00	249.21	6422.30	285.27	45.16	8.24
LRC107	3554.00	278.03	5526.53	258.46	35.69	7.04
LRC108	3365.50	220.90	5984.56	201.49	43.76	8.79
Average	3171.66	758.89	5700.88	708.33	44.16	9.69

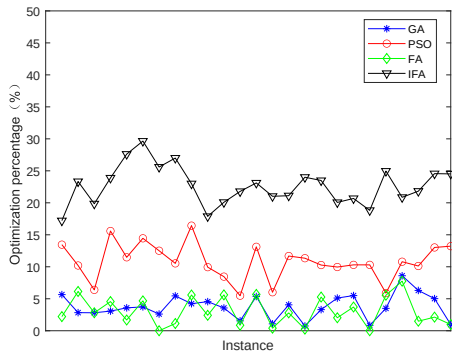


Figure 6: iter=10

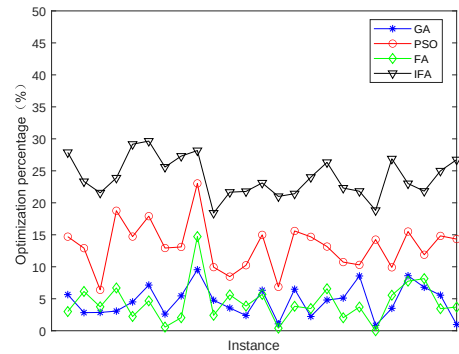


Figure 7: iter=20

Table 5: Comparison of algorithms on instance *LC*

Istance	GA		PSO		FA		IFA
	best	Gap(%)	best	Gap(%)	best	Gap(%)	best
LC101	6331.19	31.64	5704.83	15.69	6452.73	25.47	4809.50
LC102	5718.36	26.73	5045.88	10.58	5523.90	18.32	4512.08
LC103	4980.70	25.58	4634.59	14.42	4935.62	19.64	3966.24
LC104	4089.39	24.15	3517.91	6.37	4042.36	18.52	3293.81
LC105	6210.97	36.68	5450.73	16.63	6331.04	28.23	4544.01
LC106	6057.98	37.79	5184.75	15.20	6101.75	27.95	4396.54
LC107	5955.48	31.92	5247.79	13.97	5927.22	23.83	4514.59
LC108	5832.33	34.79	5214.20	17.02	5954.12	27.33	4326.94
LC109	5497.37	30.30	5051.37	16.48	5581.01	24.40	4219.00
Average	5630.42	31.07	5005.79	14.04	5649.97	23.74	4286.97

Table 6: Comparison of algorithms on instance *LR*

Istance	GA		PSO		FA		IFA
	best	Gap(%)	best	Gap(%)	best	Gap(%)	best
LR101	4978.34	17.46	4217.94	2.58	4660.31	11.82	4109.27
LR102	4560.17	14.39	4328.00	9.80	4729.11	17.45	3903.94
LR103	4297.38	18.69	3959.21	11.74	4210.18	11.70	3494.39
LR104	3955.69	21.22	3586.58	13.11	3896.11	20.02	3116.30
LR105	4704.26	15.85	4208.66	5.94	4709.50	15.95	3958.52
LR106	4373.14	19.80	3929.82	10.75	4455.539	21.28	3507.23
LR107	3938.72	15.97	3545.61	6.65	4051.14	18.30	3309.90
LR108	3820.95	20.95	3411.92	11.48	3754.73	19.56	3020.33
LR109	4507.91	20.31	3986.83	9.89	4555.57	21.14	3592.36
Average	4348.51	18.29	3908.29	9.10	4335.80	18.60	3556.92

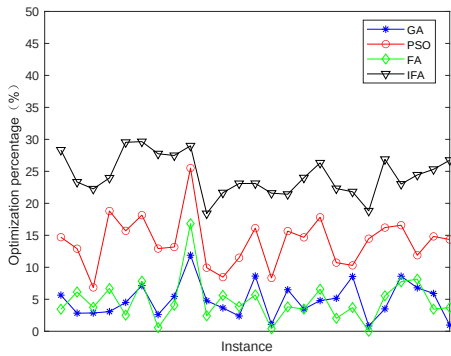


Figure 8: iter=30

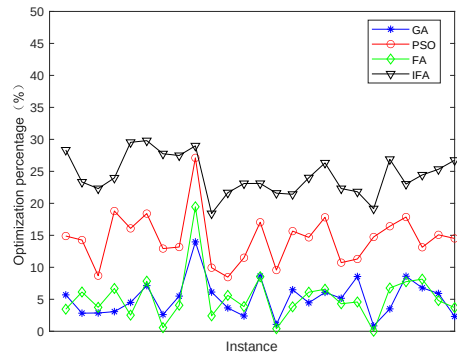


Figure 9: iter=40

Table 7: Comparison of algorithms on instance *LRC*

Istance	GA		PSO		FA		IFA
	best	Gap(%)	best	Gap(%)	best	Gap(%)	best
LRC101	5256.18	18.08	4947.74	12.97	5304.17	18.82	4305.95
LRC102	5078.19	14.49	4902.23	11.42	5237.71	17.05	4342.60
LRC103	4926.98	18.47	4236.36	5.18	4910.32	18.19	4017.02
LRC104	4622.61	24.23	3294.70	10.75	4466.22	21.58	3502.61
LRC105	5109.93	15.76	4544.10	5.28	5157.14	16.54	4304.40
LRC106	5144.56	18.98	4767.02	12.56	5069.44	17.78	4168.18
LRC107	4972.44	22.93	4481.90	14.50	5030.82	23.83	3832.03
LRC108	4781.83	25.00	4185.35	14.31	4591.76	21.89	3586.40
Average	4986.59	19.74	4498.67	10.87	4970.95	19.46	4007.40

most instances IFA can obtain the optimal solution at iteration 10. The optimal solution obtained by IFA optimizes the initial target value of up to 29.65% and an average of 21.80%. In most instances, GA and FA can also obtain the optimal solution at iteration 10. However, the optimal solutions obtained by the two algorithms can optimize 8.60% and 7.75% of the initial objective values, and on average they can optimize 3.62% and 2.96%, respectively. The comparison illustrates that IFA is less likely to be trapped in local optimum compared to GA and FA.

Figure 9 investigates that PSO can obtain results closer to IFA compared to GA and FA at iteration 10. However, most of the optimal solutions of PSO on instances are obtained at iterations of 10 to 30, while IFA is at iteration 10. The comparison illustrates the stronger global convergence of IFA compared to PSO. In addition, the final best solution obtained through IFA optimized the initial target value of 22.55% on average, while the solution obtained through PSO optimizes the average of 14.07%. The comparison shows that IFA is less likely to fall into local optimization than PSO.

The comparison results show that the Improved Firefly algorithm outperforms the other three algorithms in terms of convergence ability. IFA is more difficult to fall into local solutions than GA, PSO, and FA.

6. Conclusion

Fuel consumption of logistics vehicles is one of the main sources of carbon emissions. In recent years, fuel emission from logistics vehicles has been a topic of concern for many researchers. This paper presents a GVRPPD model and develops solutions to help enterprises develop green path plans. The model minimizes carbon economic costs while maximizing customer satisfaction by considering soft time windows constraints, customer satisfaction, the fuel consumption function, and vehicle load. We also improved the firefly algorithm to enhance the adoption of this model. A series of computational tests are carried out on different datasets to verify the effectiveness of the model and algorithm. The performance of the algorithm is verified by comparing various algorithms.

This study has three contributions. Firstly, this study establishes a GVRPPD model and evaluates the trade-off between customer satisfaction and fuel consumption in GVRP considering vehicle load. Secondly, by introducing chaotic mapping and differential evolution to deal with GVRPPD, the firefly algorithm is improved. Third, through the statistical analysis of the design experiment, it is found that GVRPPD can significantly reduce carbon emissions without significantly reducing customer satisfaction, which provides an opportunity for logistics enterprises to develop green operations.

Future research may be extended to the mixed fleet of various vehicles with different parameters. Further research may also consider extending the model to the dynamic nature of the business. In this case, dynamically arriving orders can be added to the routing decision dynamically.

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Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Authors' contributions

All authors contributed to the study conception and design. Data collection and analysis were performed by Qian Zhou. The first draft of the manuscript was written by Qian Zhou and all authors commented on previous versions of the manuscript. Review & editing of draft were performed by Zhibin Wu. All authors read and approved the final manuscript.

References

- Aktar, M. S., De, M., Mazumder, S. K., & Maiti, M. (2022). Multi-objective green 4-dimensional transportation problems for damageable items through type-2 fuzzy random goal programming. *Applied Soft Computing*, 130, 109681. doi:<https://doi.org/10.1016/j.asoc.2022.109681>.
- Ali, I. M., Essam, D., & Kasmarik, K. (2019). A novel differential evolution mapping technique for generic combinatorial optimization problems. *Applied Soft Computing*, 80, 297–309. doi:<https://doi.org/10.1016/j.asoc.2019.04.017>.
- Barth, M., & Boriboonsomsin, K. (2008). Real-world co2 impacts of traffic congestion transportation research record. *Journal of the Transportation Research Board*, 2058, 163–171. doi:<https://doi.org/10.3141/2058-20>.
- Behnamian, J., Ghadimi, M., & Farajiamiri, M. (2023). Data mining-based firefly algorithm for green vehicle routing problem with heterogeneous fleet and refueling constraint. *Artificial Intelligence Review*, 56, 6557–6589. doi:<https://doi.org/10.1007/s10462-022-10336-9>.
- Bruglieri, M., Ferone, D., Festa, P., & Pisacane, O. (2022). A grasp with penalty objective function for the green vehicle routing problem with private capacitated stations. *Computers & Operations Research*, 143, 105770. doi:<https://doi.org/10.1016/j.cor.2022.105770>.
- Bruglieri, M., Mancini, S., Pezzella, F., & Pisacane, O. (2019). A path-based solution approach for the green vehicle routing problem. *Computers & Operations Research*, 103, 109–122. doi:<https://doi.org/10.1016/j.cor.2018.10.019>.
- Chen, H.-K., Chou, H.-W., Hsueh, C.-F., & Yu, Y.-J. (2015). The paired many-to-many pickup and delivery problem: an application. *Top*, 23, 220–243. doi:<https://doi.org/10.1007/s11750-014-0335-y>.
- Dell'Amico, M., Righini, G., & Salani, M. (2006). A branch-and-price approach to the vehicle routing problem with simultaneous distribution and collection. *Transportation Science*, 40, 235–247. doi:<https://doi.org/10.1287/trsc.1050.0118>.
- Elgharably, N., Easa, S., Nassef, A., & El Damatty, A. (2023). Stochastic multi-objective vehicle routing model in green environment with customer satisfaction. *IEEE Transactions on Intelligent Transportation Systems*, 24, 1337–1355. doi:[10.1109/TITS.2022.3156685](https://doi.org/10.1109/TITS.2022.3156685).
- EPA (2020). Sources of greenhouse gas emissions. <https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions>.

- Erdoğan, S., & Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation research part E: logistics and transportation review*, 48, 100–114. doi:<https://doi.org/10.1016/j.tre.2011.08.001>.
- Fan, H., Zhang, Y., Tian, P., Lv, Y., & Fan, H. (2021). Time-dependent multi-depot green vehicle routing problem with time windows considering temporal-spatial distance. *Computers & Operations Research*, 129, 105211. doi:<https://doi.org/10.1016/j.cor.2021.105211>.
- Foroutan, R. A., Rezaeian, J., & Mahdavi, I. (2020). Green vehicle routing and scheduling problem with heterogeneous fleet including reverse logistics in the form of collecting returned goods. *Applied Soft Computing*, 94, 106462. doi:<https://doi.org/10.1016/j.asoc.2020.106462>.
- Gao, Y., Wu, H., & Wang, W. (2023). A hybrid ant colony optimization with fireworks algorithm to solve capacitated vehicle routing problem. *Applied Intelligence*, 53, 7326–7342. doi:<https://doi.org/10.1007/s10489-022-03912-7>.
- HadjTaieb, S., Hani, Y., Loukil, T. M., & El Mhamedi, A. (2022). Green vrp applied to home health care problem. *IFAC-PapersOnLine*, 55, 3154–3159. doi:<https://doi.org/10.1016/j.ifacol.2022.10.214>.
- Ibrahim, M., Putri, M., & Utama, D. M. (2020). A literature review on reducing carbon emission from supply chain system: drivers, barriers, performance indicators, and practices. In *IOP Conference Series: Materials Science and Engineering* (p. 012034). IOP Publishing volume 722.
- Kaur, M., Singh, D., Sun, K., & Rawat, U. (2020). Color image encryption using non-dominated sorting genetic algorithm with local chaotic search based 5d chaotic map. *Future Generation Computer Systems*, 107, 333–350. doi:<https://doi.org/10.1016/j.future.2020.02.029>.
- Koyuncu, I., & Yavuz, M. (2019). Duplicating nodes or arcs in green vehicle routing: A computational comparison of two formulations. *Transportation Research Part E: Logistics and Transportation Review*, 122, 605–623. doi:<https://doi.org/10.1016/j.tre.2018.11.003>.
- Küçükoğlu, İ., Ene, S., Aksoy, A., & Öztürk, N. (2015). A memory structure adapted simulated annealing algorithm for a green vehicle routing problem. *Environmental Science and Pollution Research*, 22, 3279–3297. doi:<https://doi.org/10.1007/s11356-014-3253-5>.
- Kuo, Y. (2010). Using simulated annealing to minimize fuel consumption for the time-dependent vehicle routing problem. *Computers & Industrial Engineering*, 59, 157–165. doi:<https://doi.org/10.1016/j.cie.2010.03.012>.
- Li, H., & Lim, A. (2003). A metaheuristic for the pickup and delivery problem with time windows. *International Journal on Artificial Intelligence Tools*, 12, 173–186. doi:<https://doi.org/10.1109/ICTAI.2001.974461>.
- Li, J., & Zhang, J. (2014). Study on the effect of carbon emission trading mechanism on logistics distribution routing decisions. *Systems Engineering-Theory & Practice*, 34, 1779–1787.
- Li, Y., Han, M., & Guo, Q. (2020). Modified whale optimization algorithm based on tent chaotic mapping and its application in structural optimization. *KSCE Journal of Civil Engineering*, 24, 3703–3713. doi:<https://doi.org/10.1007/s12205-020-0504-5>.
- Lin, C., Choy, K. L., Ho, G. T., & Ng, T. W. (2014). A genetic algorithm-based optimization model for supporting green transportation operations. *Expert Systems with Applications*, 41, 3284–3296. doi:<https://doi.org/10.1016/j.eswa.2013.11.032>.
- Liu, Y., Yu, Y., Zhang, Y., Baldacci, R., Tang, J., Luo, X., & Sun, W. (2023). Branch-cut-and-price for the time-dependent green vehicle routing problem with time windows. *INFORMS Journal on Computing*, 35, 14–30. doi:<https://doi.org/10.1287/ijoc.2022.1195>.
- Luo, H., Dridi, M., & Grunder, O. (2023). A branch-price-and-cut algorithm for a time-dependent green vehicle routing problem with the consideration of traffic congestion. *Computers & Industrial Engineering*, 177, 109093. doi:<https://doi.org/10.1016/j.cie.2023.109093>.
- Madankumar, S., & Rajendran, C. (2018). Mathematical models for green vehicle routing problems with pickup and delivery: A case of semiconductor supply chain. *Computers & Operations Research*, 89, 183–192. doi:<https://doi.org/10.1016/j.cor.2016.03.013>.
- Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations Research*, 43, 264–281. doi:<https://doi.org/10.1287/opre.43.2.264>.
- Nagy, G., & Salhi, S. (2005). Heuristic algorithms for single and multiple depot vehicle routing problems with pickups and deliveries. *European Journal of Operational Research*, 162, 126–141. doi:<https://doi.org/10.1016/j.ejor.2002.11.003>.
- Olivera, A. C., García-Nieto, J. M., & Alba, E. (2015). Reducing vehicle emissions and fuel consumption in the city by using particle swarm optimization. *Applied Intelligence*, 42, 389–405. doi:<https://doi.org/10.1007/s10489-014-0604-3>.
- Poonthalir, G., & Nadarajan, R. (2018). A fuel efficient green vehicle routing problem with varying speed constraint (f-gvrp). *Expert Systems with Applications*, 100, 131–144. doi:<https://doi.org/10.1016/j.eswa.2018.01.052>.
- Poonthalir, G., Nadarajan, R., & Geetha, S. (2015). Vehicle routing problem with limited refueling halts using particle swarm optimization with greedy mutation operator. *RAIRO-Operations Research*, 49, 689–716. doi:<https://doi.org/10.1051/ro/2014064>.

- Rieck, J., Ehrenberg, C., & Zimmermann, J. (2014). Many-to-many location-routing with inter-hub transport and multi-commodity pickup-and-delivery. *European Journal of Operational Research*, 236, 863–878. doi:<https://doi.org/10.1016/j.ejor.2013.12.021>.
- Sadati, M. E. H., & Çatay, B. (2021). A hybrid variable neighborhood search approach for the multi-depot green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 149, 102293. doi:<https://doi.org/10.1016/j.tre.2021.102293>.
- Salhi, S., & Nagy, G. (1999). A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. *Journal of the operational Research Society*, 50, 1034–1042. doi:<https://doi.org/10.1057/palgrave.jors.2600808>.
- Scora, G., & Barth, M. (2006). Comprehensive modal emissions model (cmem). *User guide. Centre for environmental research and technology. University of California, Riverside*, 1070, 1580.
- Storn, R., & Price, K. (1997). Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11, 341.
- Subramanian, A., Uchoa, E., Pessoa, A. A., & Ochi, L. S. (2013). Branch-cut-and-price for the vehicle routing problem with simultaneous pickup and delivery. *Optimization Letters*, 7, 1569–1581. doi:<https://doi.org/10.1007/s11590-012-0570-9>.
- Vincent, F. Y., Jodiawan, P., & Gunawan, A. (2021). An adaptive large neighborhood search for the green mixed fleet vehicle routing problem with realistic energy consumption and partial recharges. *Applied Soft Computing*, 105, 107251. doi:<https://doi.org/10.1016/j.asoc.2021.107251>.
- Wang, H., Li, M., Wang, Z., Li, W., Hou, T., Yang, X., Zhao, Z., & Sun, T. (2022). Heterogeneous fleets for green vehicle routing problem with traffic restrictions. *IEEE Transactions on Intelligent Transportation Systems*, . doi:[10.1109/TITS.2022.3197424](https://doi.org/10.1109/TITS.2022.3197424).
- Xiao, Y., Zhao, Q., Kaku, I., & Xu, Y. (2012). Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & operations research*, 39, 1419–1431. doi:<https://doi.org/10.1016/j.cor.2011.08.013>.
- Xu, X., & Wei, Z. (2023). Dynamic pickup and delivery problem with transshipments and lifo constraints. *Computers & Industrial Engineering*, 175, 108835. doi:<https://doi.org/10.1016/j.cie.2022.108835>.
- Xu, Z., Elomri, A., Pokharel, S., & Mutlu, F. (2019). A model for capacitated green vehicle routing problem with the time-varying vehicle speed and soft time windows. *Computers & Industrial Engineering*, 137, 106011. doi:[A model for capacitated green vehicle routing problem with the time-varying vehicle speed and soft time windows](https://doi.org/10.1016/j.cie.2019.106011).
- Yang, X.-S. (2009). Firefly algorithms for multimodal optimization. In *Stochastic Algorithms: Foundations and Applications: 5th International Symposium, SAGA 2009, Sapporo, Japan, October 26-28, 2009. Proceedings 5* (pp. 169–178). Springer.
- Yanik, S., Bozkaya, B., & deKervenoael, R. (2014). A new vrppd model and a hybrid heuristic solution approach for e-tailing. *European Journal of Operational Research*, 236, 879–890.
- Yu, S., Zhu, S., Ma, Y., & Mao, D. (2015). A variable step size firefly algorithm for numerical optimization. *Applied Mathematics and Computation*, 263, 214–220.
- Yu, Y., Wang, S., Wang, J., & Huang, M. (2019). A branch-and-price algorithm for the heterogeneous fleet green vehicle routing problem with time windows. *Transportation Research Part B: Methodological*, 122, 511–527. doi:<https://doi.org/10.1016/j.trb.2019.03.009>.